Soft photon QED corrections to $B \to K\ell^+\ell^-$

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Outline

- Introduction
- Matrix elements
- Objectives
- $\mathcal{O}(\alpha)$ QED corrections
- Results
- Summary and conclusion

Introduction

- Flavour changing neutral currents (FCNCs) are both loop and CKM suppressed

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- $B \to K\ell^+\ell^-$ allow to test the lepton flavour universality (LFU)[1] :

$$R_K^{\mu e} \equiv \frac{\int_{1GeV^2}^{6GeV^2} dq^2 \frac{d\Gamma(B^0 \to K^0\mu^+\mu^-)}{dq^2}}{\int_{1GeV^2}^{6GeV^2} dq^2 \frac{d\Gamma(B^0 \to K^0 e^+ e^-)}{dq^2}} \qquad R_K^{\mu e}|_{SM} = 1.00 \pm 0.01 \qquad R_K^{\mu e}|_{exp} = 0.846^{+0.060+0.016}_{-0.054-0.014}$$

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QED effects...?

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Matrix elements

- The effective Hamiltonian for $b \to s\ell^+\ell^-$ transition: $H_{eff} = 4\frac{G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^{10} C_i(\mu)\mathcal{O}_i(\mu)$

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$$O_7 = -\frac{e}{16\pi^2}\frac{2m_b}{q} i(\bar{s}\sigma_{\mu\nu}q^\nu Rb)(\bar{l}\gamma^\mu l),$$

$$O_9 = \frac{e^2}{16\pi^2}(\bar{s}\gamma_\mu Lb)(\bar{l}\gamma^\mu l),$$

$$O_{10} = \frac{e^2}{16\pi^2}(\bar{s}\gamma_\mu Lb)(\bar{l}\gamma^\mu\gamma_5 l).$$

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$$M_0(B \to Kl^+l^-) = \frac{G_F\alpha}{2\sqrt{2}\pi} V_{ts}^* V_{tb} \left[\left(\left\{ C_9^{eff} f_+ + C_7^{eff} \frac{2f_T m_b}{m_B + m_k} \right\} p^\mu + \left\{ C_9^{eff} f_- - C_7^{eff} \frac{2f_T m_b}{q^2} (m_B - m_k) \right\} q^\mu \right) \left(\bar{l}\gamma_\mu l \right) - \left(C_{10} f_+ p^\mu + C_{10} f_- q^\mu \right) \left(\bar{l}\gamma_\mu\gamma_5 l \right) \right]$$

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$$\tilde{M} = -e\epsilon_\alpha(k)\bar{u}(p_2)\Gamma_A^\mu\frac{(\gamma_\mu p_3^\mu + \gamma_\mu k^\mu) - m_l}{2p_3 \,.\, k}\gamma^\alpha v(p_3) \otimes H_{A\mu}(p_0,p_1) + e\epsilon_\alpha(k)\bar{u}(p_2)\gamma^\alpha\frac{(\gamma_\mu p_2^\mu + \gamma_\mu k^\mu) + m_l}{2p_2 \,.\, k}\Gamma_A^\mu v(p_3) \otimes H_{A\mu}(p_0,p_1)$$

$$+eQ_B\epsilon_\alpha(k)\frac{2p_0^\alpha}{2p_0 \,.\, k}\bar{u}(p_2)\Gamma_A^\mu v(p_3) \otimes H_{A\mu}(p_0-k,p_1) - eQ_K\epsilon_\alpha(k)\frac{2p_1^\alpha}{2p_1 \,.\, k}\bar{u}(p_2)\Gamma_A^\mu v(p_3) \otimes H_{A\mu}(p_0,p_1+k)$$

Where, $H_\mu(p_i,p_j) = f_+(p_i+p_j)_\mu + f_-(p_i-p_j)_\mu$

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Where, $H_\mu(p_i, p_j) = f_+(p_i + p_j)_\mu + f_-(p_i - p_j)_\mu$

Leptonic part	Gauge invariant
Hadronic part	Not gauge invariant
Total amplitude	Not gauge invariant

Objectives

- To fix the gauge invariance of matrix element.
- To get the $\mathcal{O}(\alpha)$ QED correction for the decay width $(B \to K\ell^+\ell^-)$ and $R_K^{\mu e}$.
- To discuss the collinear divergences and their cancellation.

$\mathcal{O}(\alpha)$ QED corrections

Real photon emission

- The addition of a CT $\left(e(Q_B + Q_K)\xi_A k_\mu \left[\bar{u}(p_2)\Gamma_A^\mu v(p_3) \right] \right)$ is required to preserve gauge invariance

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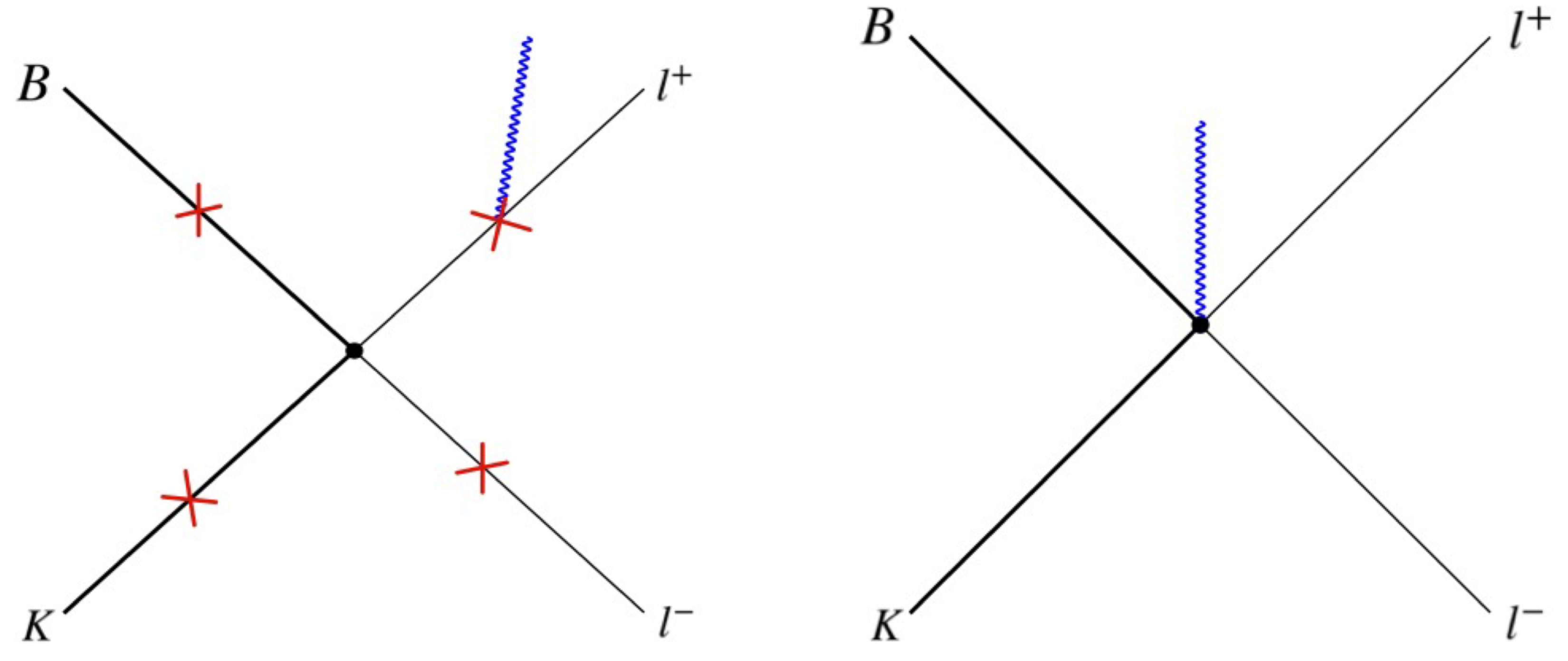

Representative diagrams of real emissions

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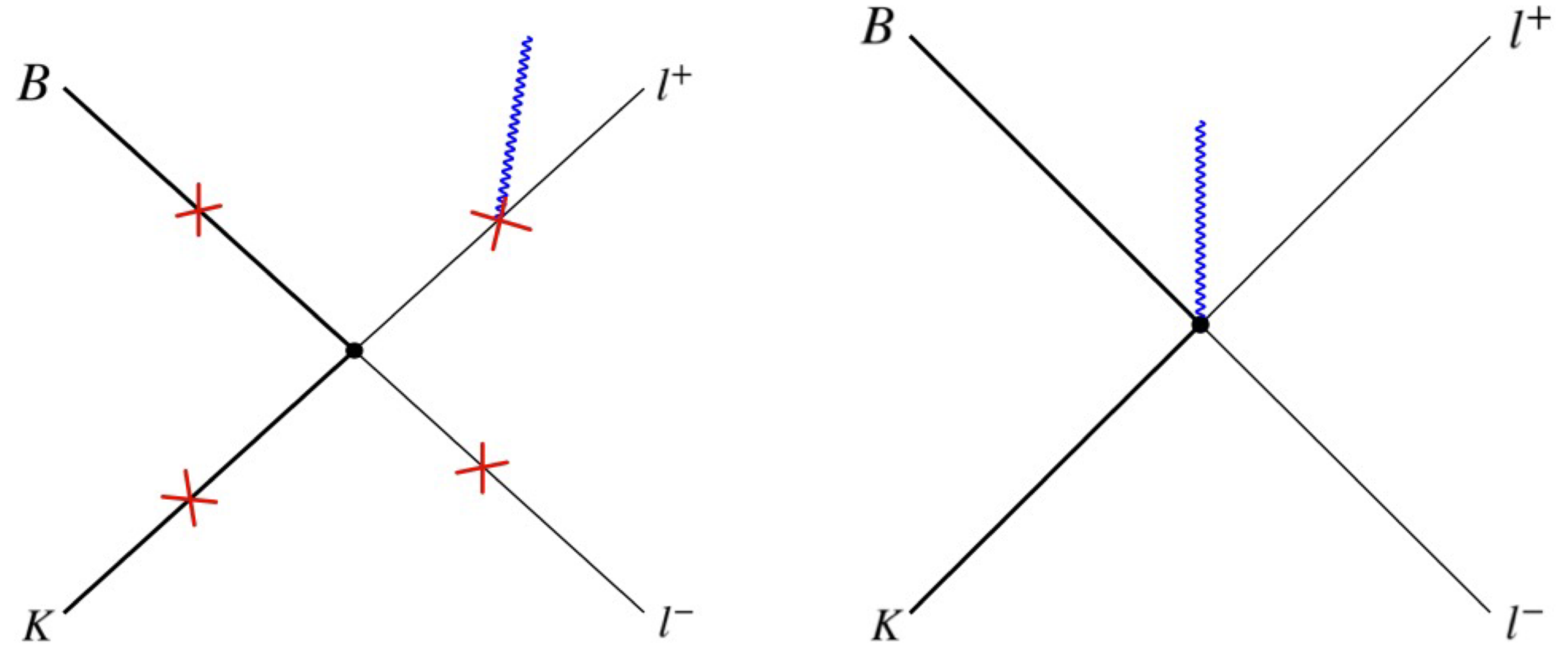

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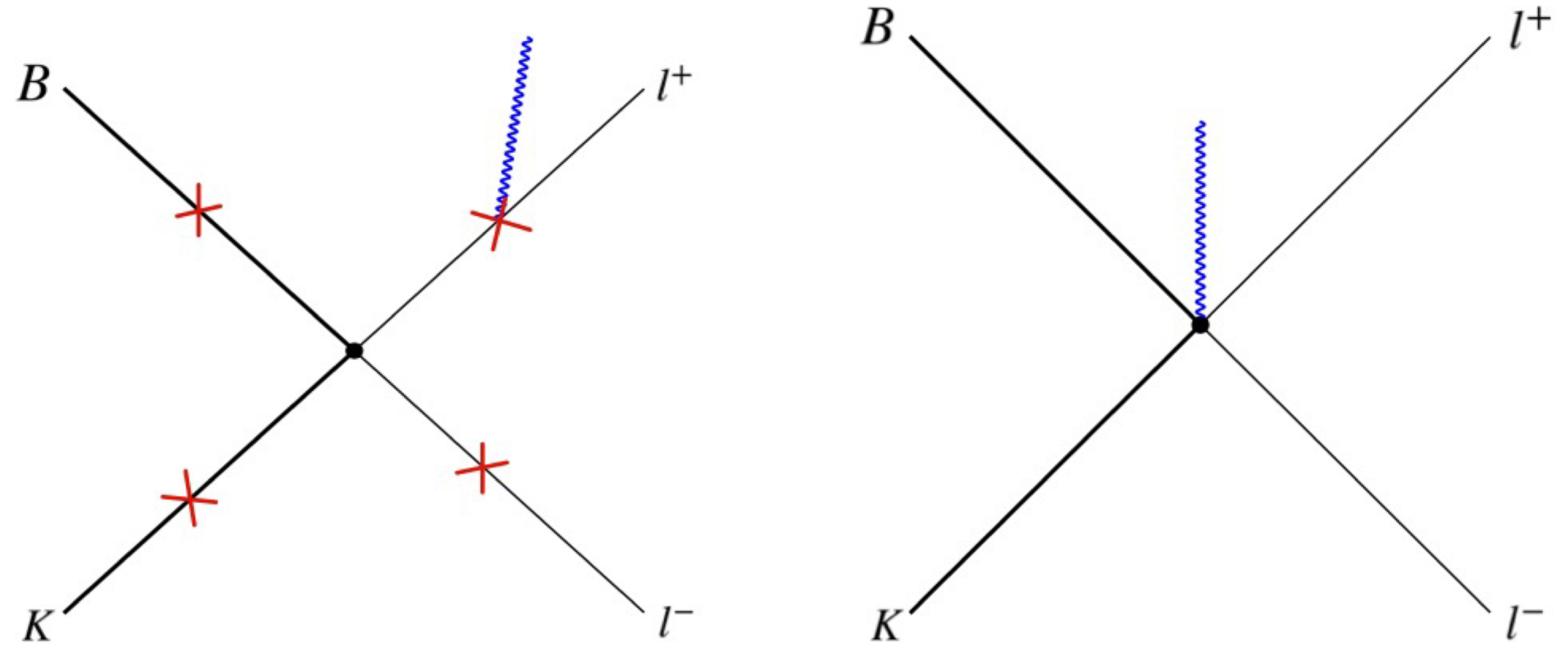
Non-radiative decay width

due to Low's term

Non-IR contribution



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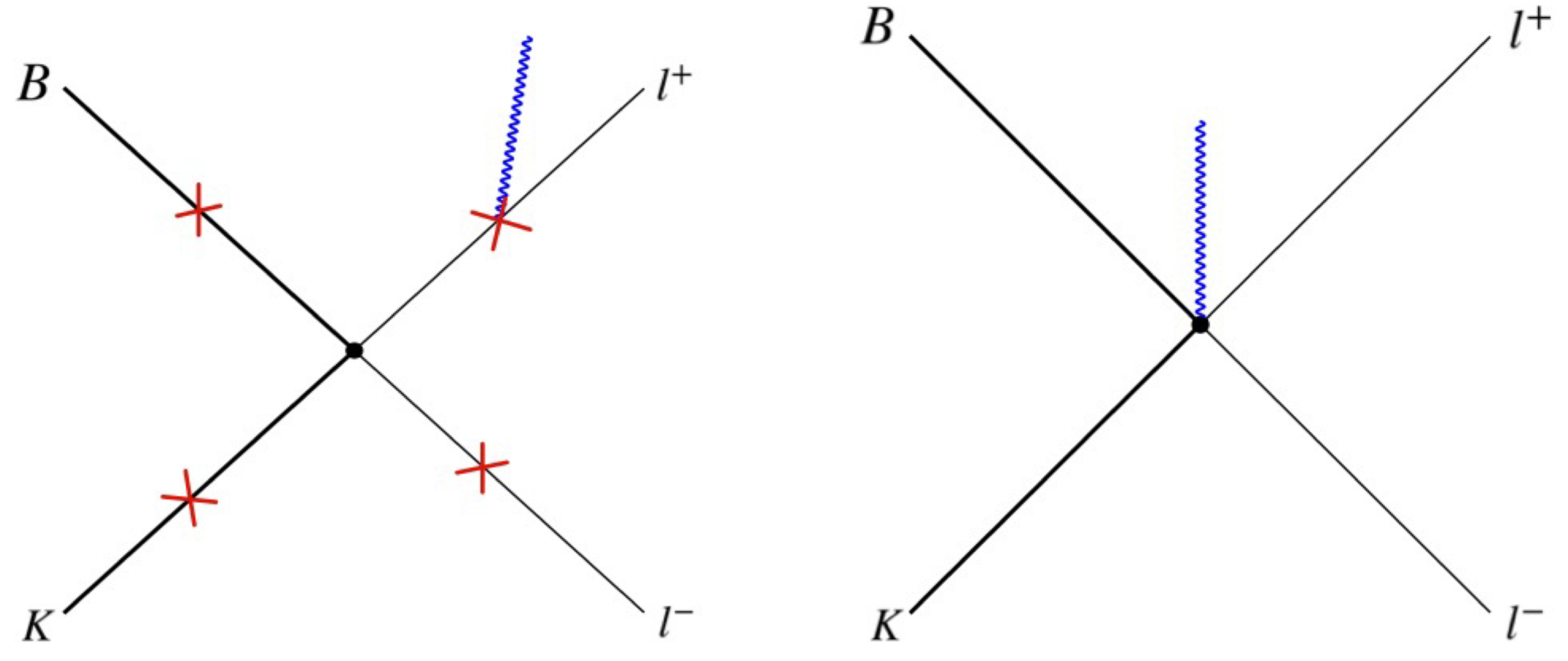

Representative diagrams of real emissions

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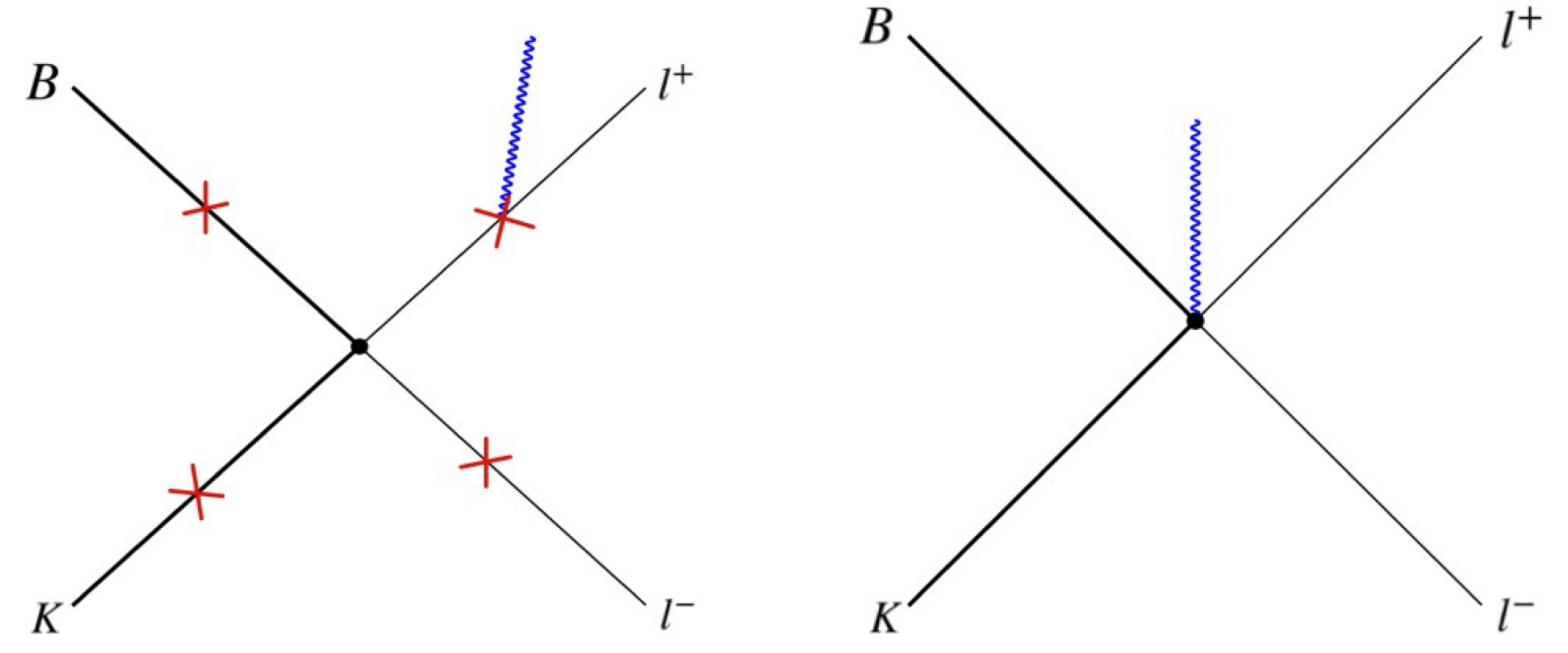


Representative diagrams of real emissions

- The non-IR term is important to see the cancellation of collinear divergences.
- Charge conservation and integrating over photons momentum k gives:

$$\tilde{B}_{ij} = \frac{Q_iQ_j\eta_i\eta_j}{2\pi}\left\{\ln\left(\frac{k^2_{max}m_im_j}{\lambda^2 E_iE_j}\right) - \frac{p_i\cdot p_j}{2}\left[\int_{-1}^{1}\frac{dx}{p_x^2}\ln\left(\frac{k^2_{max}}{E_x^2}\right) + \int_{-1}^{1}\frac{dx}{p_x^2}\ln\left(\frac{p_x^2}{\lambda^2}\right)\right]\right\}$$

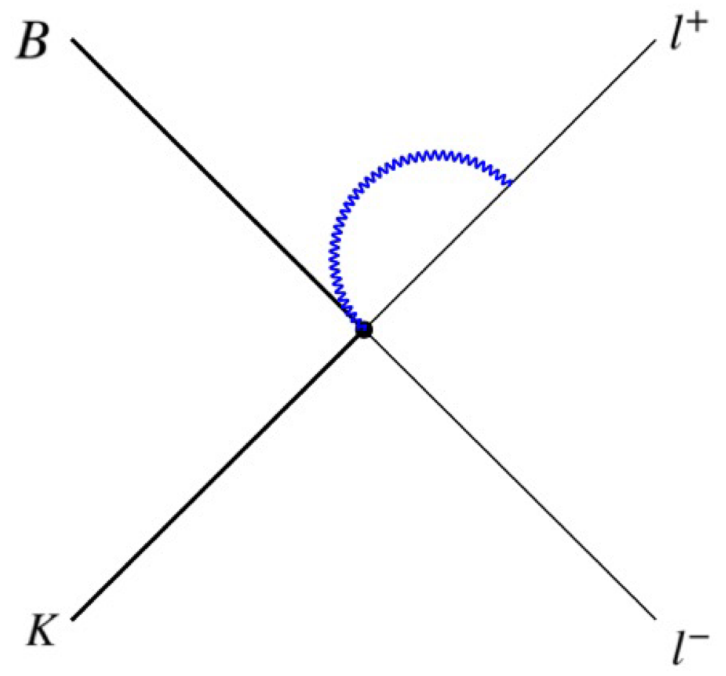
Where, $2p_x = (1+x)p_i + (1-x)p_j \qquad 2E_x = (1+x)E_i + (1-x)E_j \qquad 2p_x' = (1+x)p_i\eta_i - (1-x)p_j\eta_j$

Virtual photon corrections

- **Due to contact term:** Contain ultraviolet divergences

  $\Longrightarrow$ get cancelled for leptons but remained for charged mesons

- Our method to construct the contact term provides $\mathcal{O}(e)$ term whereas the obtained UV divergence is at $\mathcal{O}(e^2)$.



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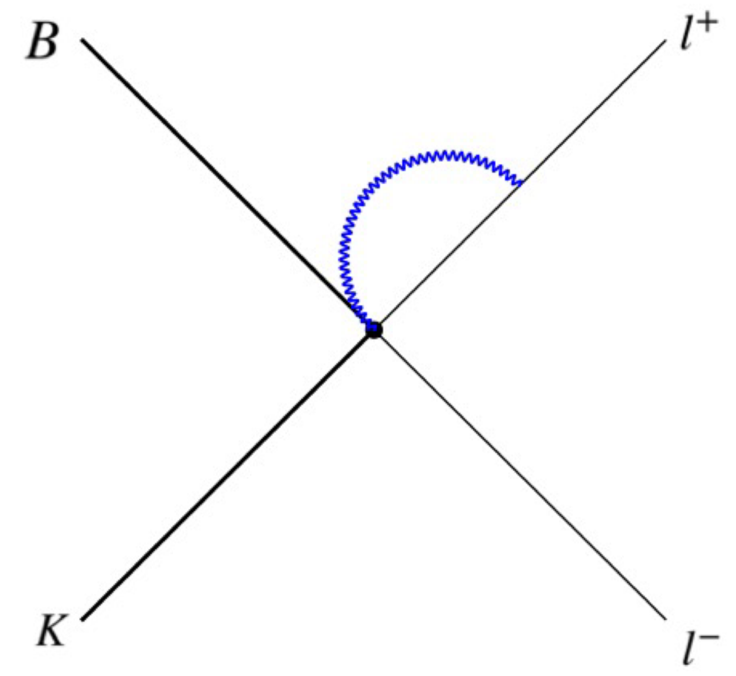

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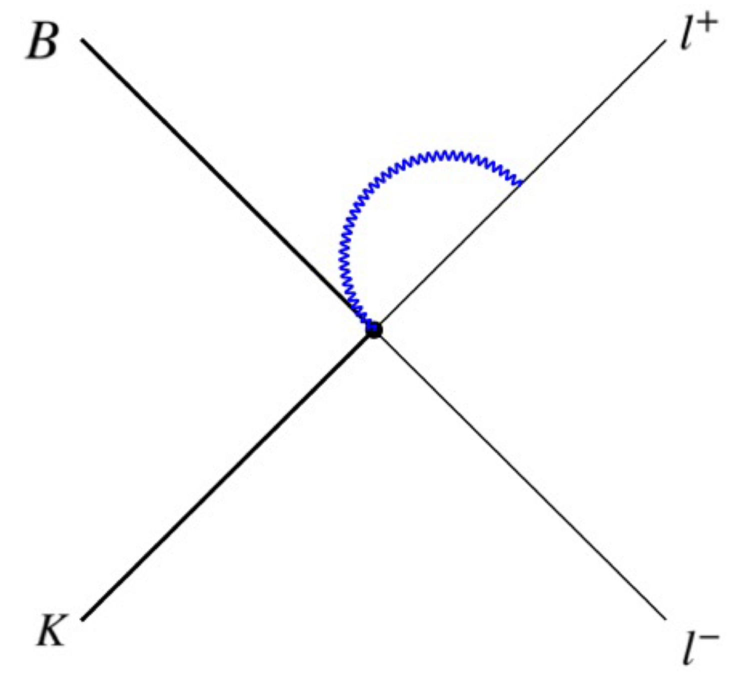

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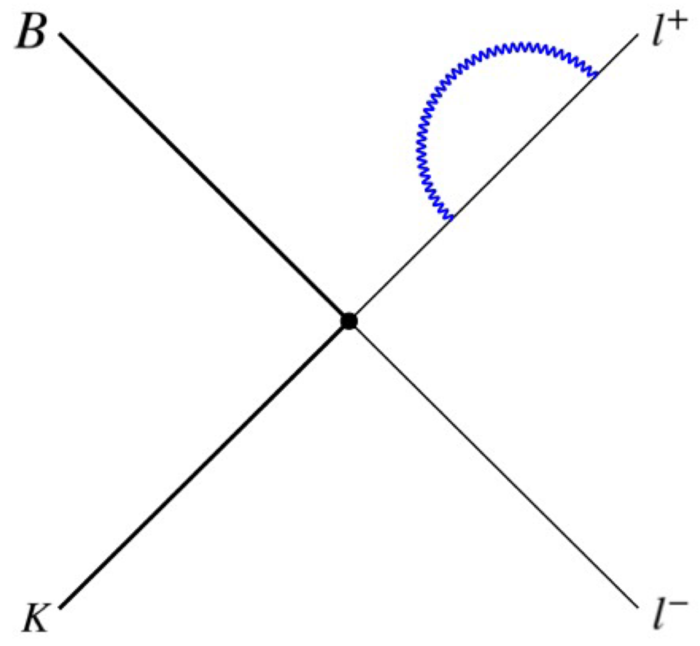
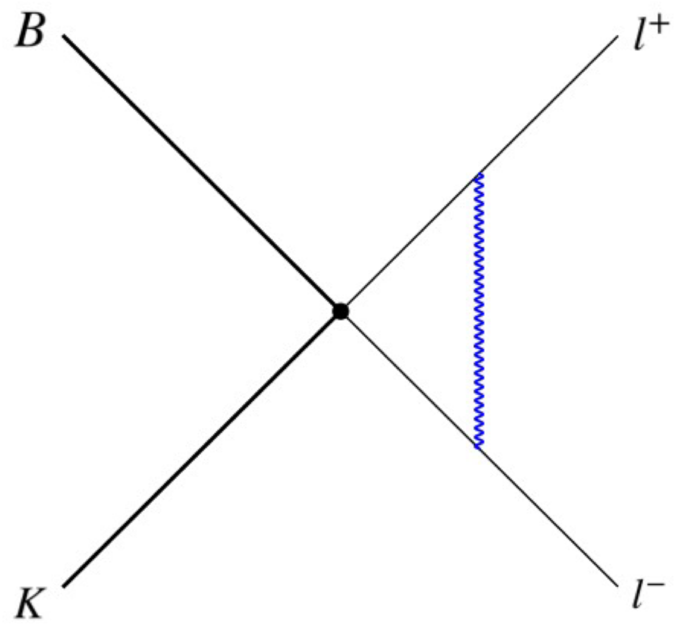
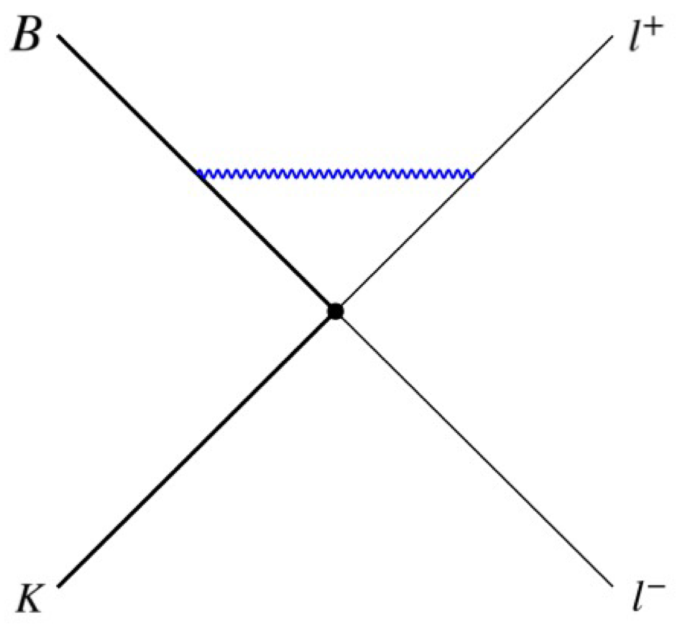
- Discarded the leftover UV divergences. The finite part is proportional to momenta of particles and numerically it contributes to $\sim 1.4\,\%$.

- Evaluating virtual diagrams: $M_{\text{virtual}} = M_0 \left[1 + \alpha B + \frac{\alpha}{2\pi} \right] + M_{CT}$







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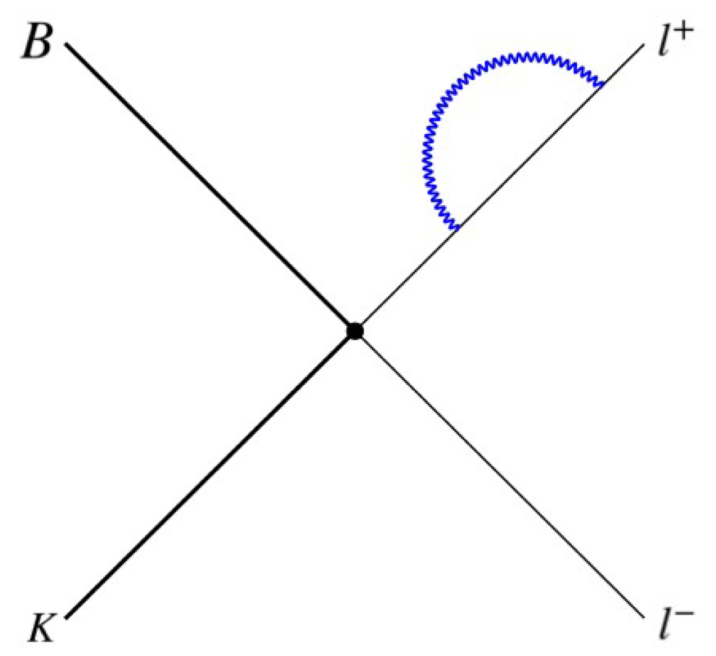
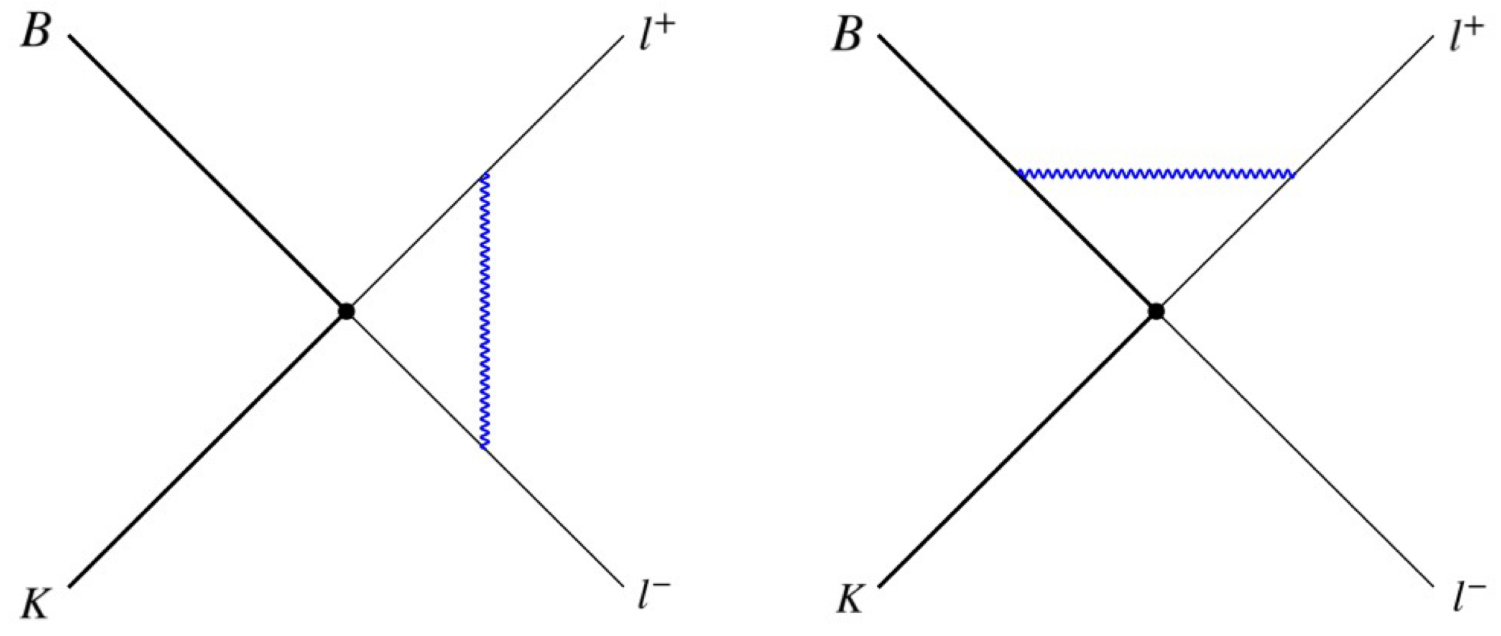


With $B_{ij} = \frac{-1}{2\pi} Q_i Q_j \eta_i \eta_j \left[ln(\frac{m_i m_j}{\lambda^2}) + \frac{1}{4}\int_{-1}^{1} dx ln(\frac{p_x'^2}{m_i m_j}) + \frac{p_i \cdot p_j \eta_i \eta_j}{2}\int_{-1}^{1}\frac{dx}{p_x'^2} ln(\frac{p_x'^2}{\lambda^2})\right]$



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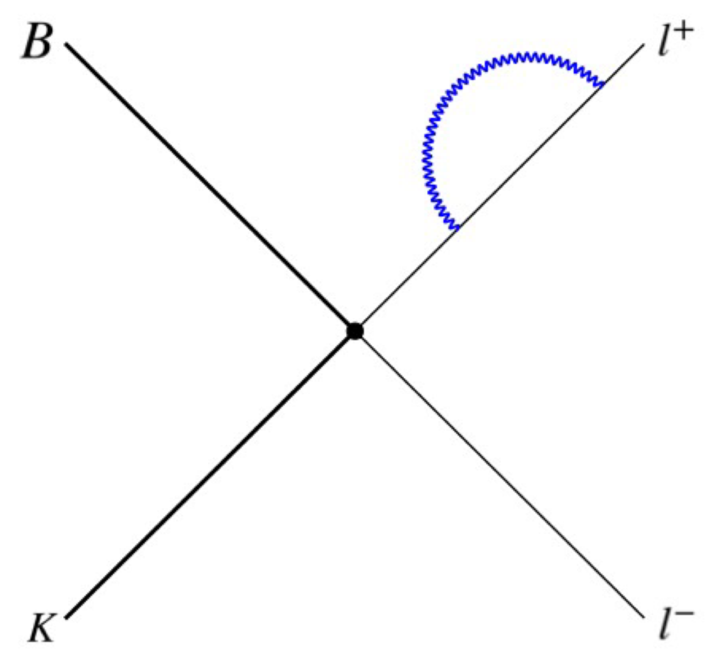

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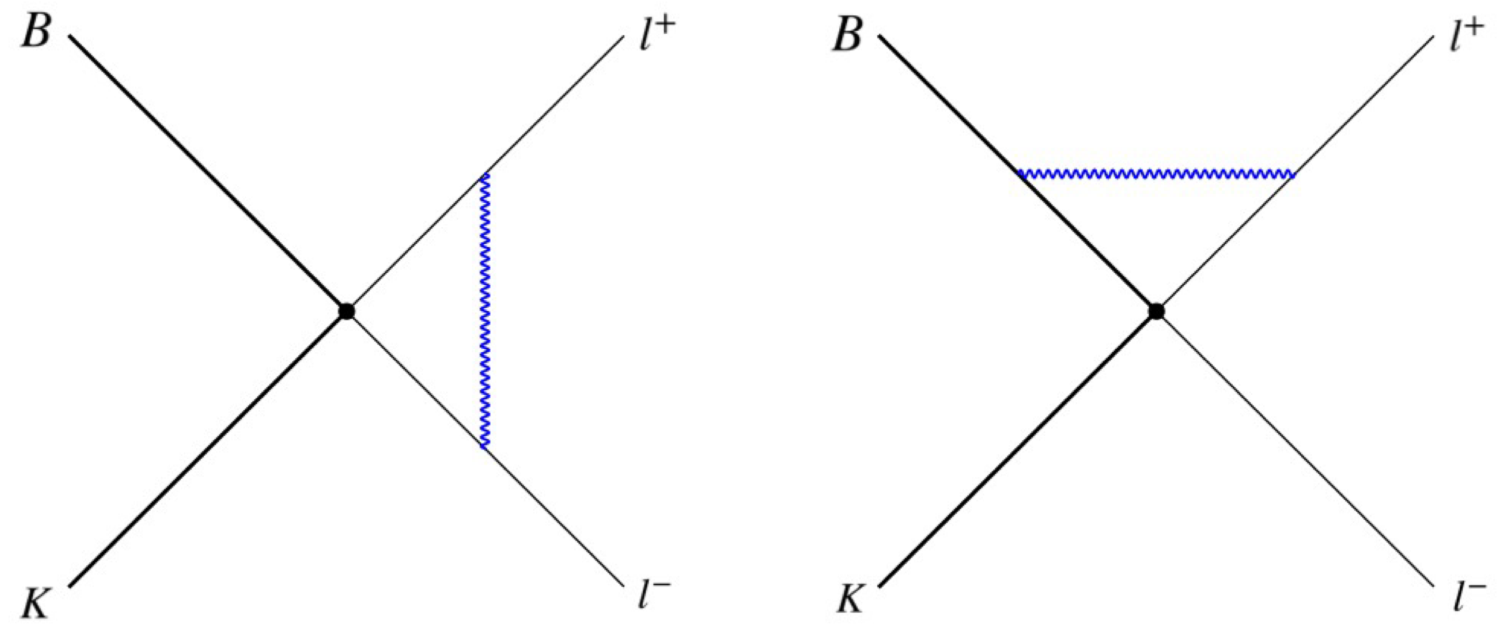
- The Coulomb factor :

$$\Omega_c = \prod_{i<j} \frac{-2\pi\alpha}{\beta_{ij}} \frac{1}{e^{\frac{-2\pi\alpha}{\beta_{ij}}} - 1}$$

where, $\beta_{ij} = \sqrt{1 - \frac{m_i^2 m_j^2}{(p_i \cdot p_j)^2}}$



- **The total decay rate:** $$d\Gamma_{real} = d\Gamma_0 \left(1 + 2\alpha \underbrace{(\tilde{\mathscr{B}} + \mathscr{B})}_{\mathscr{H}_{ij}} + \frac{\alpha}{\pi} \right) \Omega_c + d\Gamma'$$

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Coulomb correction factor

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- The correction factor $\Delta^i(\mathcal{O}(\alpha))$:

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- **The shift $\Delta_{R_K^{\mu e}}$:**

$$\Delta^i_{R_K^{\mu e}} = R_K^{0\mu e} \left(\frac{\Delta\Gamma^i_\mu}{\Gamma^i_\mu} - \frac{\Delta\Gamma^i_e}{\Gamma^i_e} \right)$$

Results

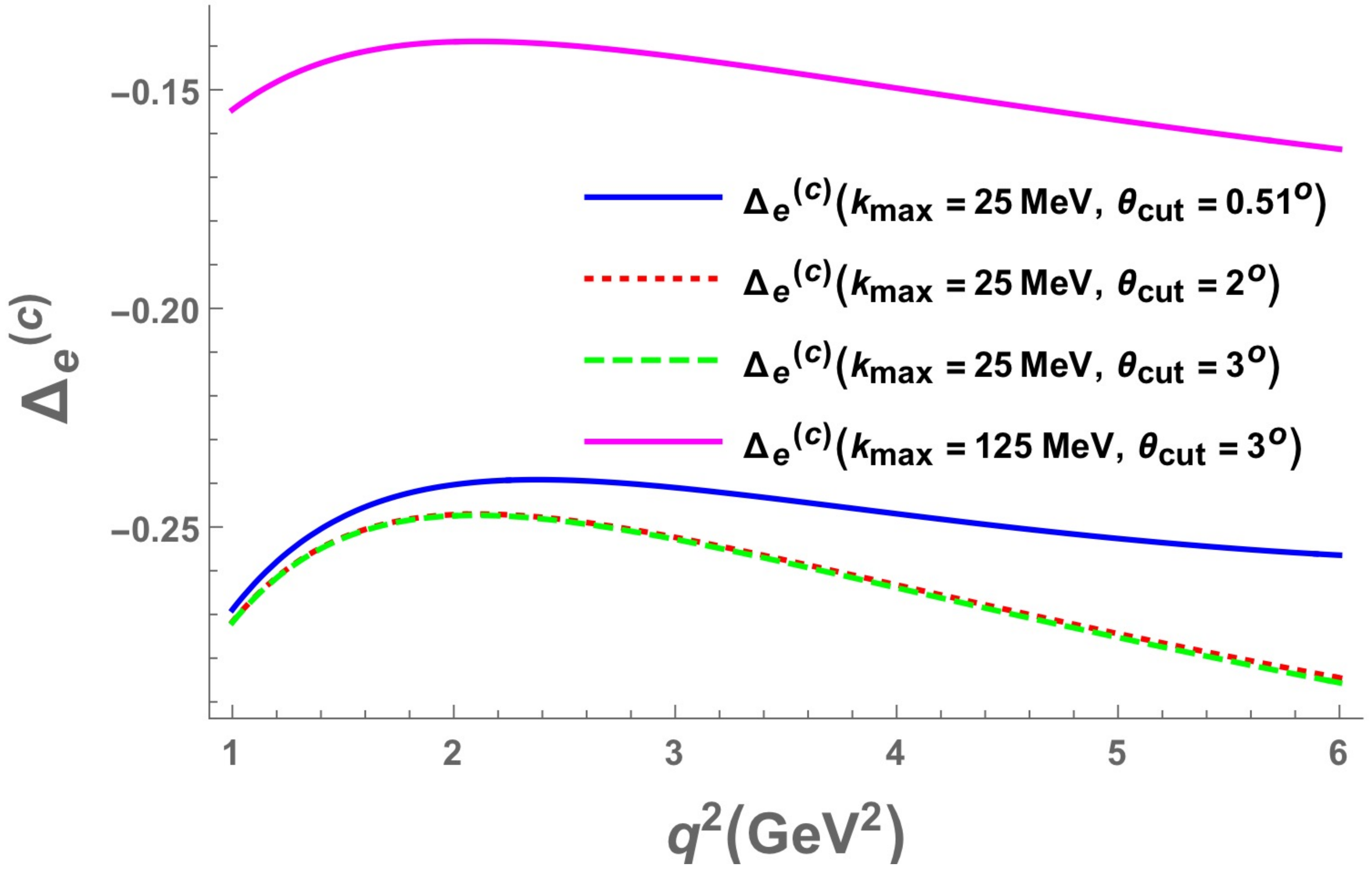


Figure 1: $\mathcal{O}(\alpha)$ corrections to charged $B \to K e^+ e^-$.

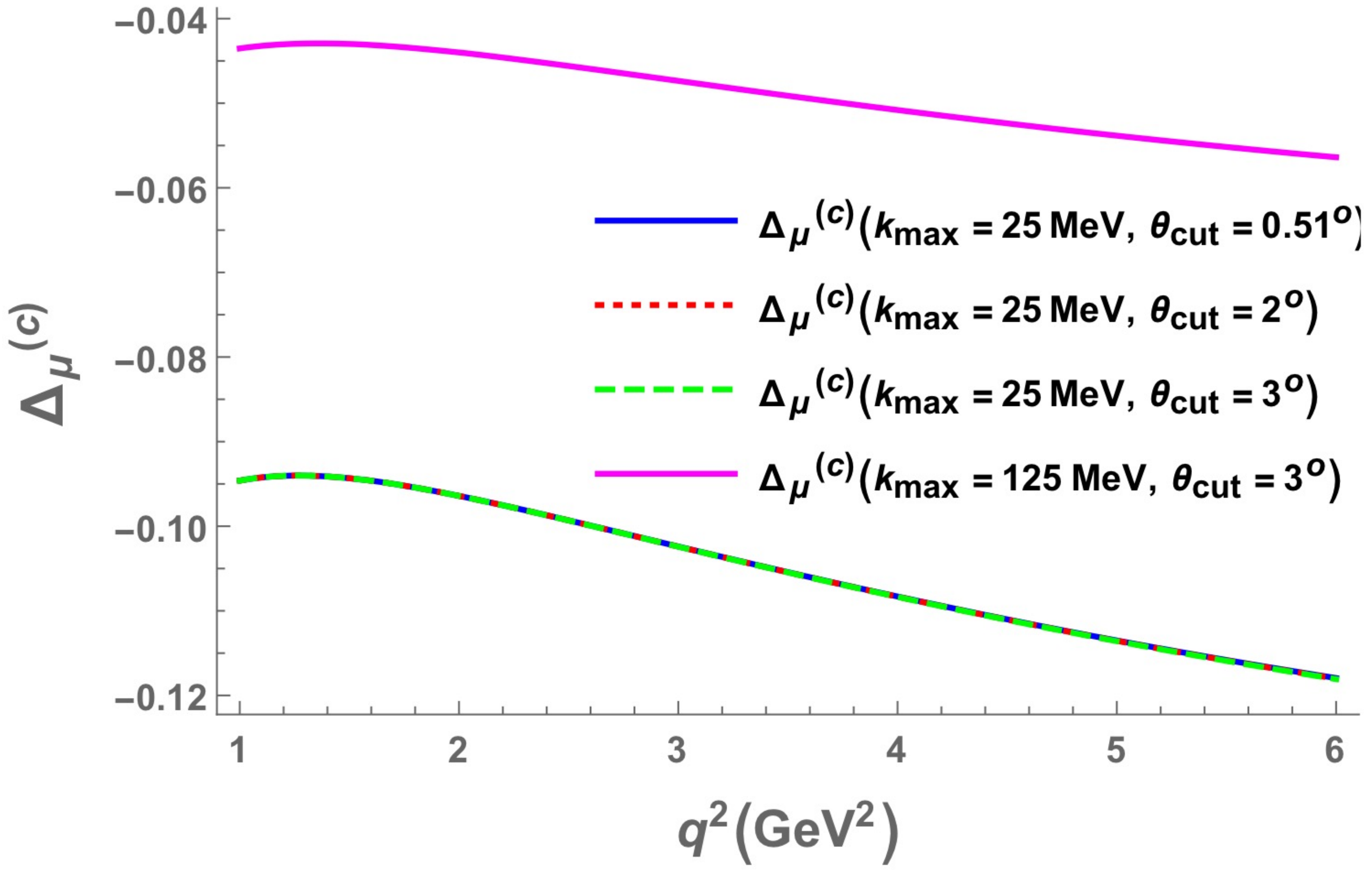
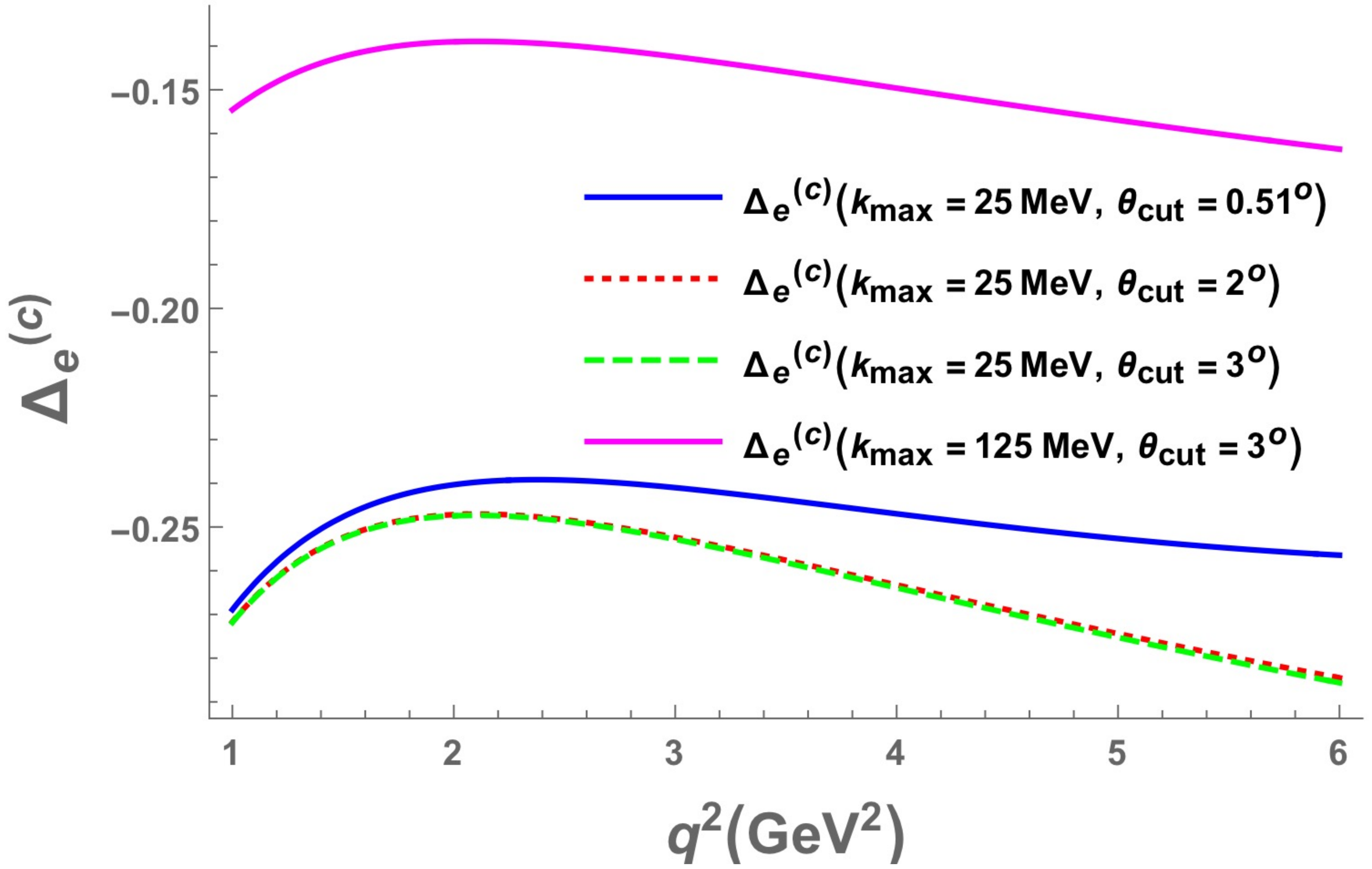


Figure 2: $\mathcal{O}(\alpha)$ corrections to charged $B \to K \mu^+ \mu^-$.

Results



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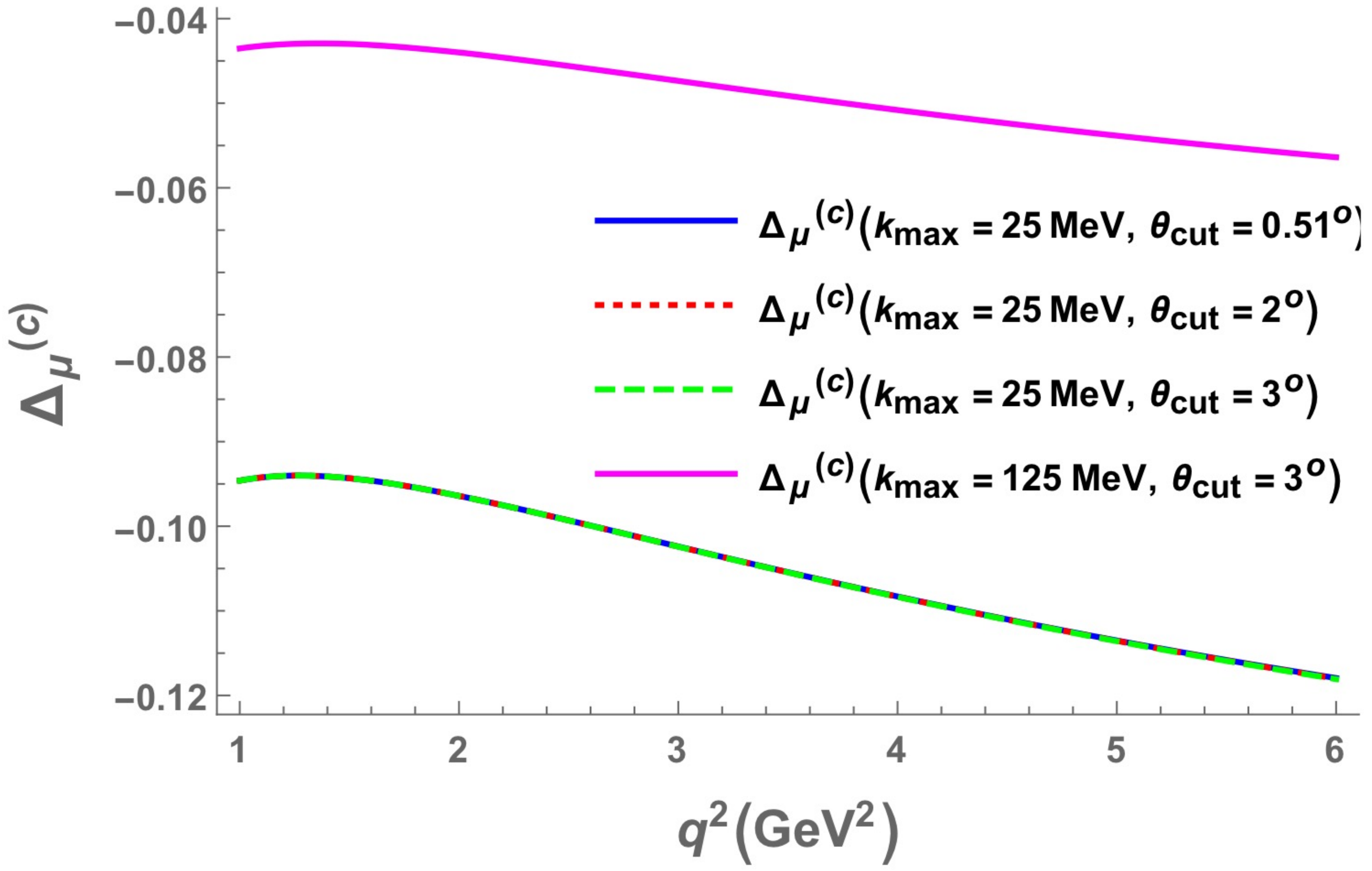

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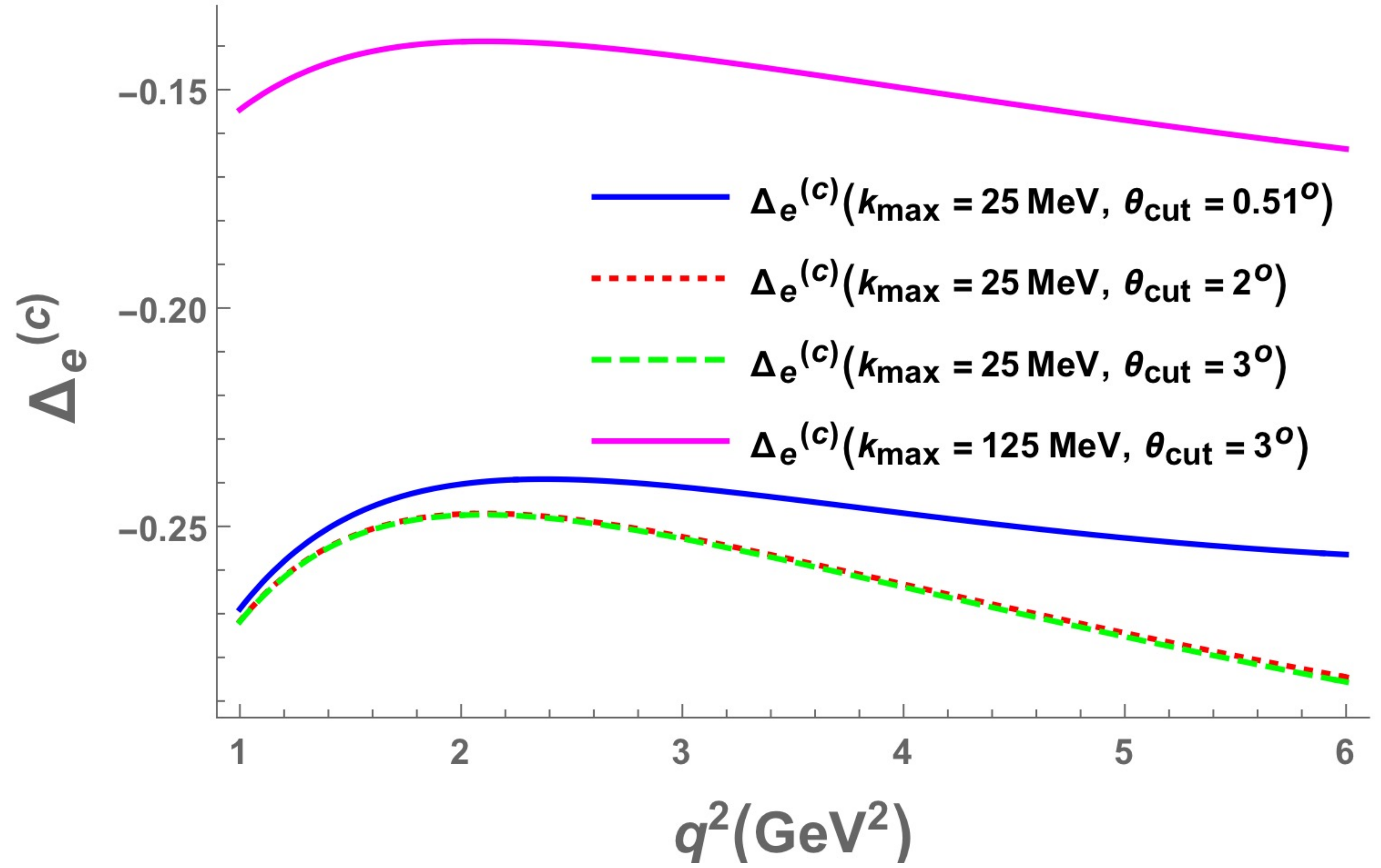

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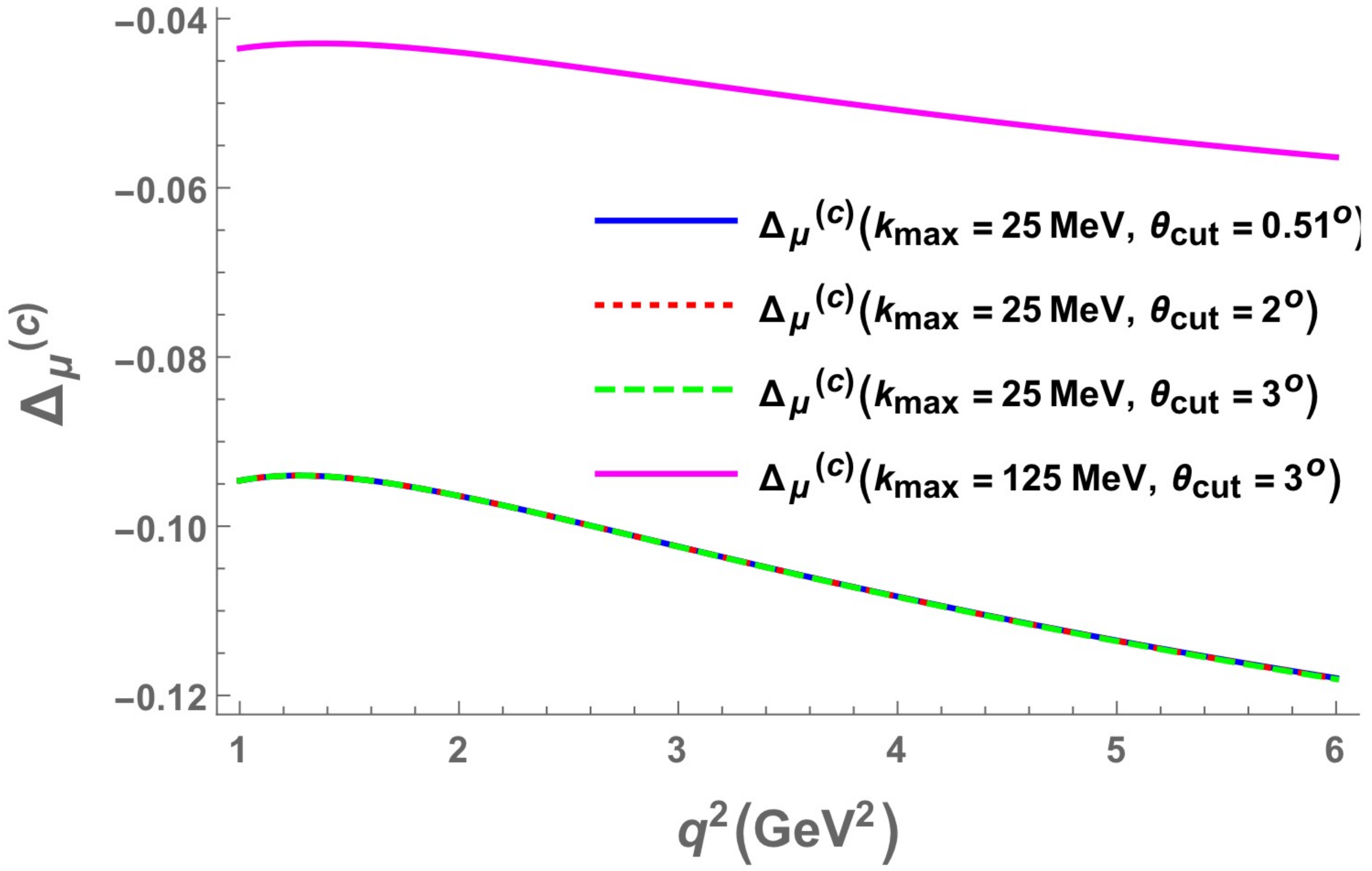

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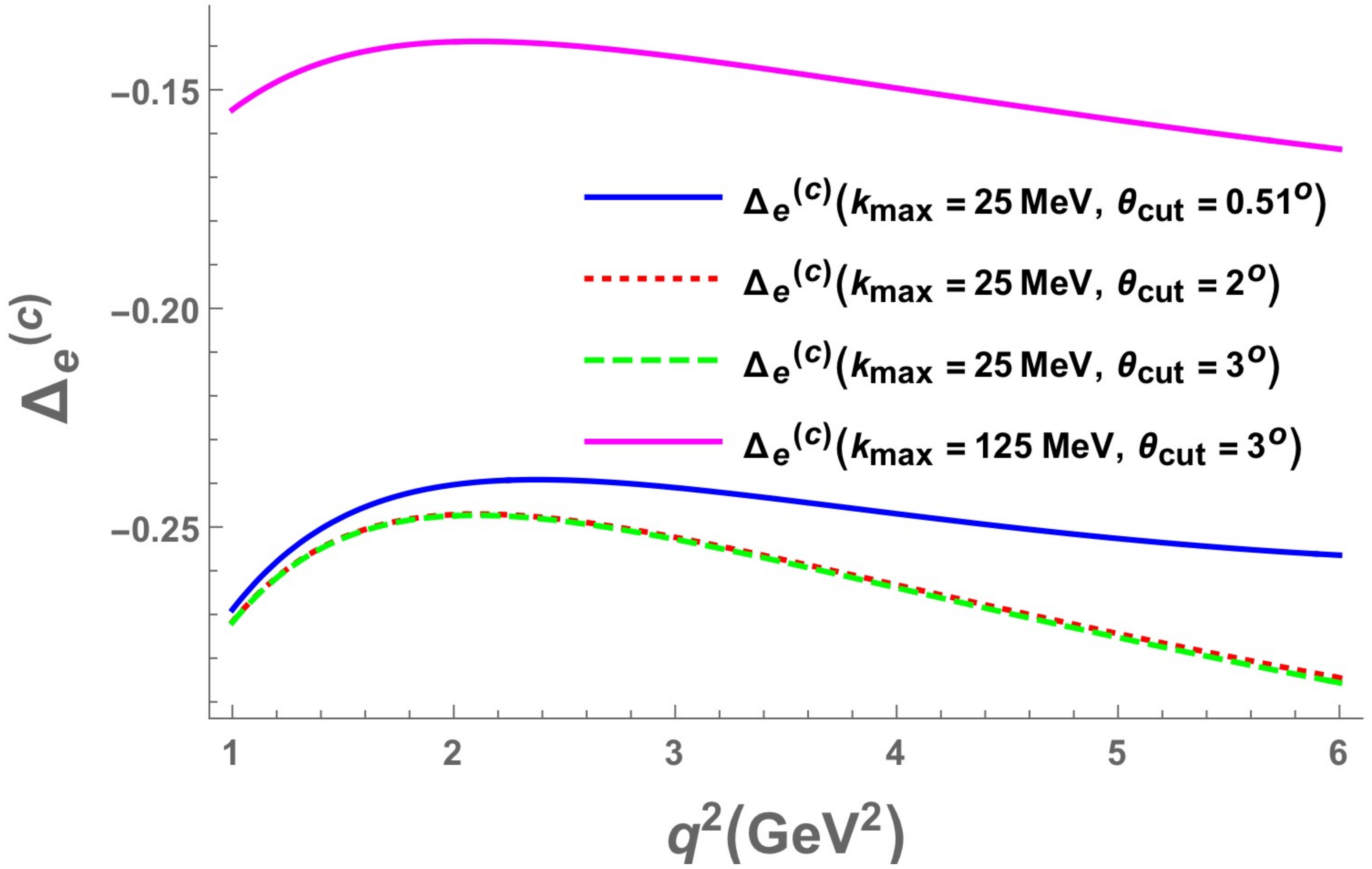

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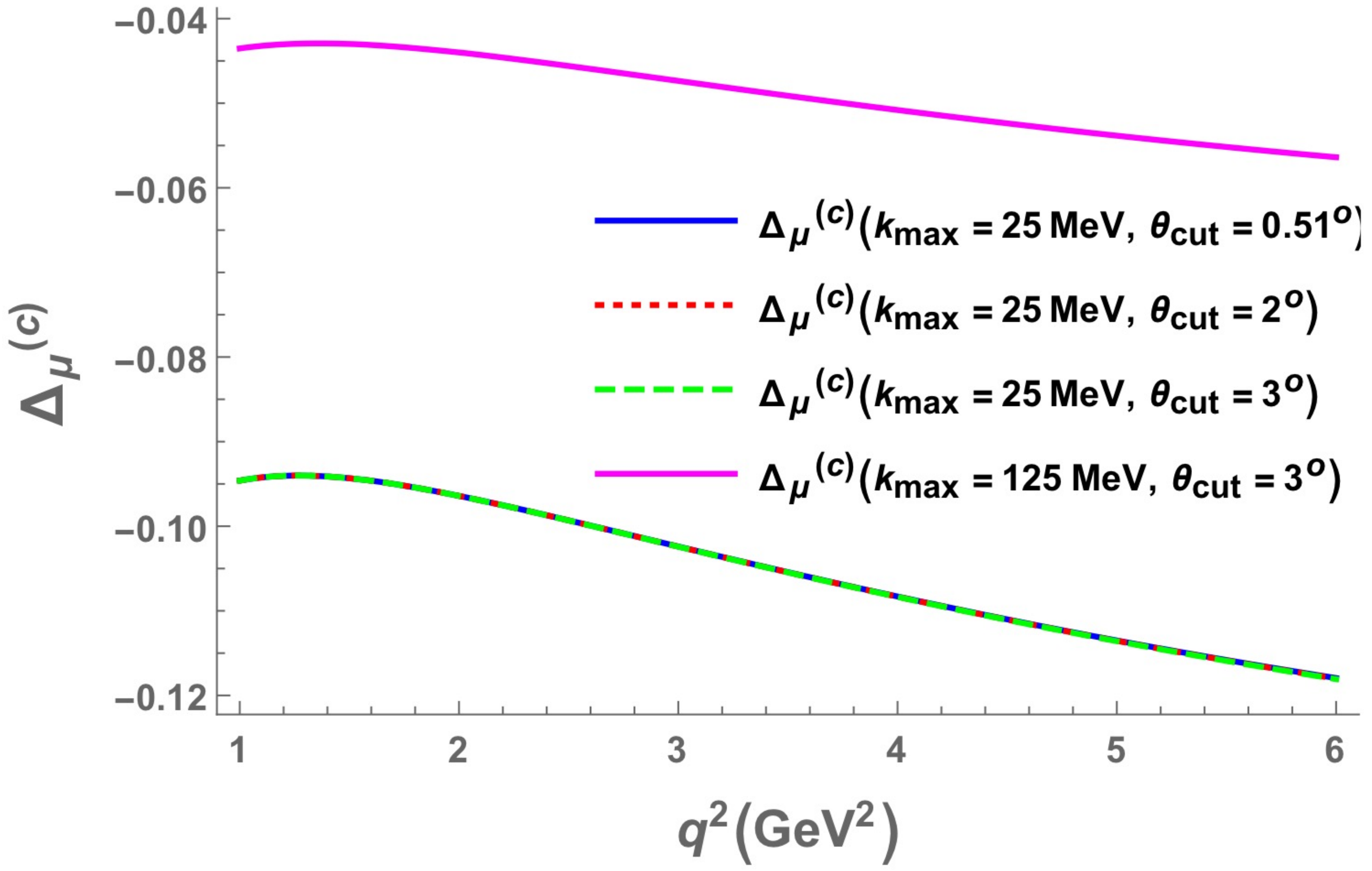

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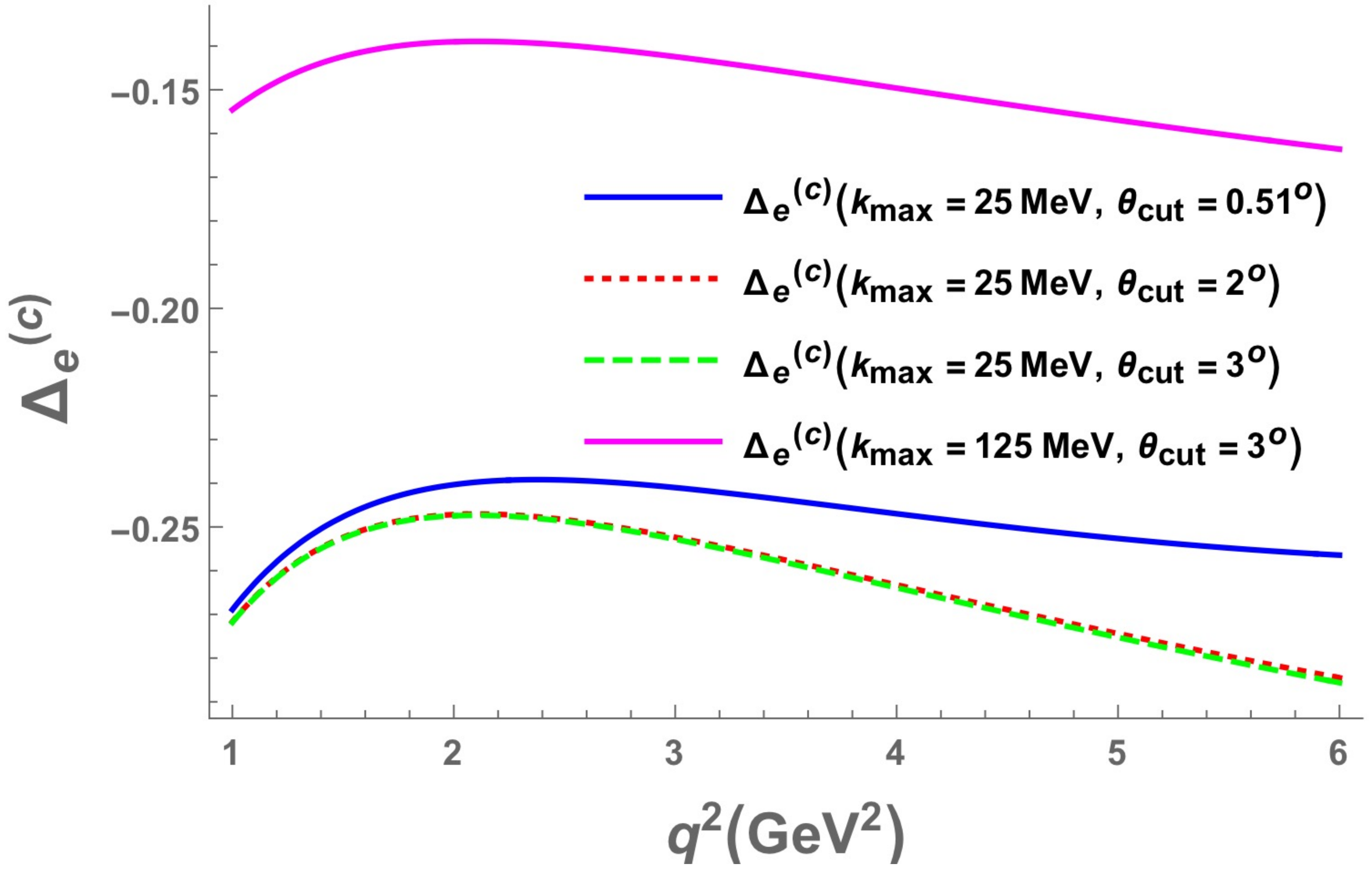

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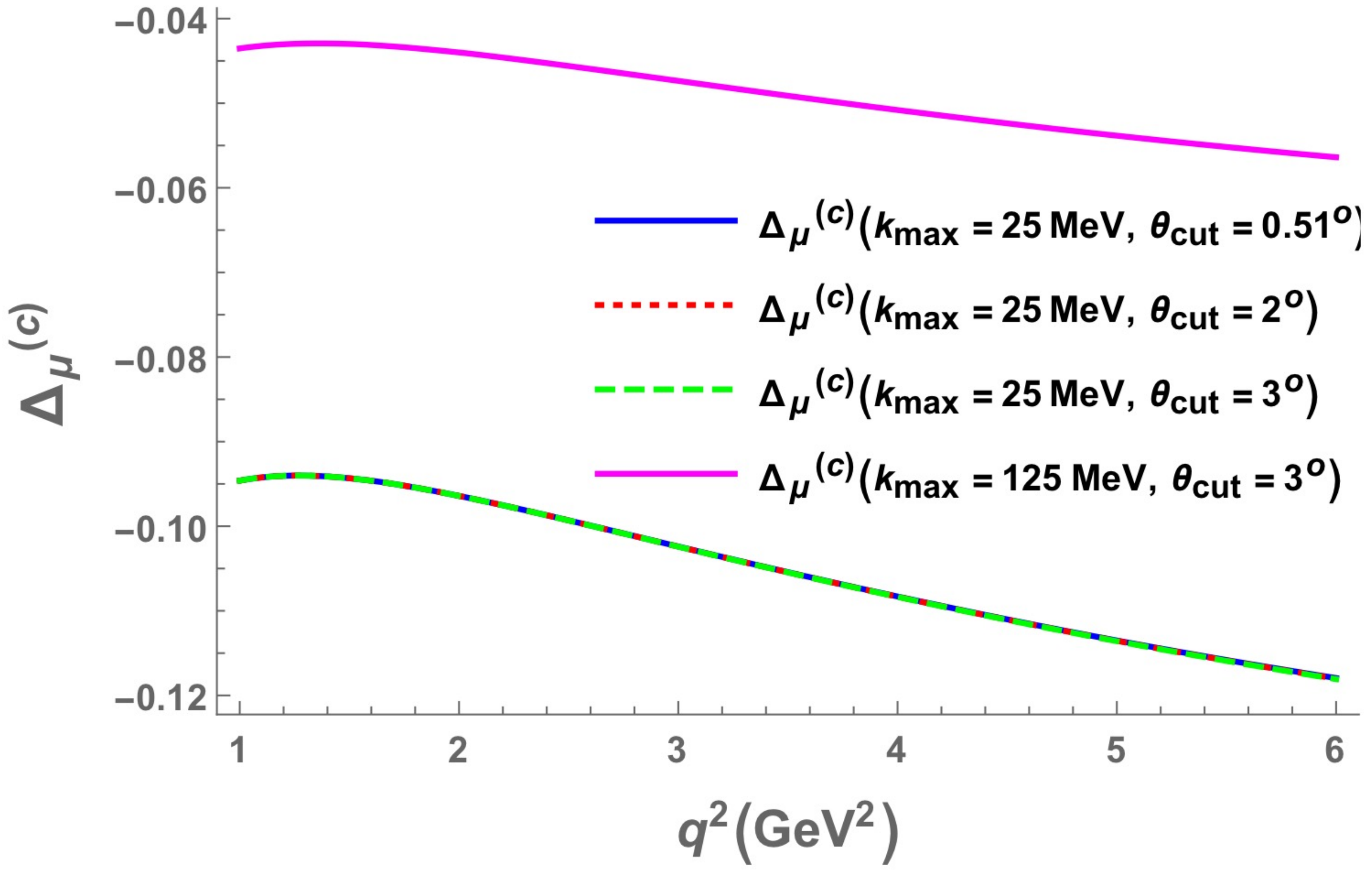

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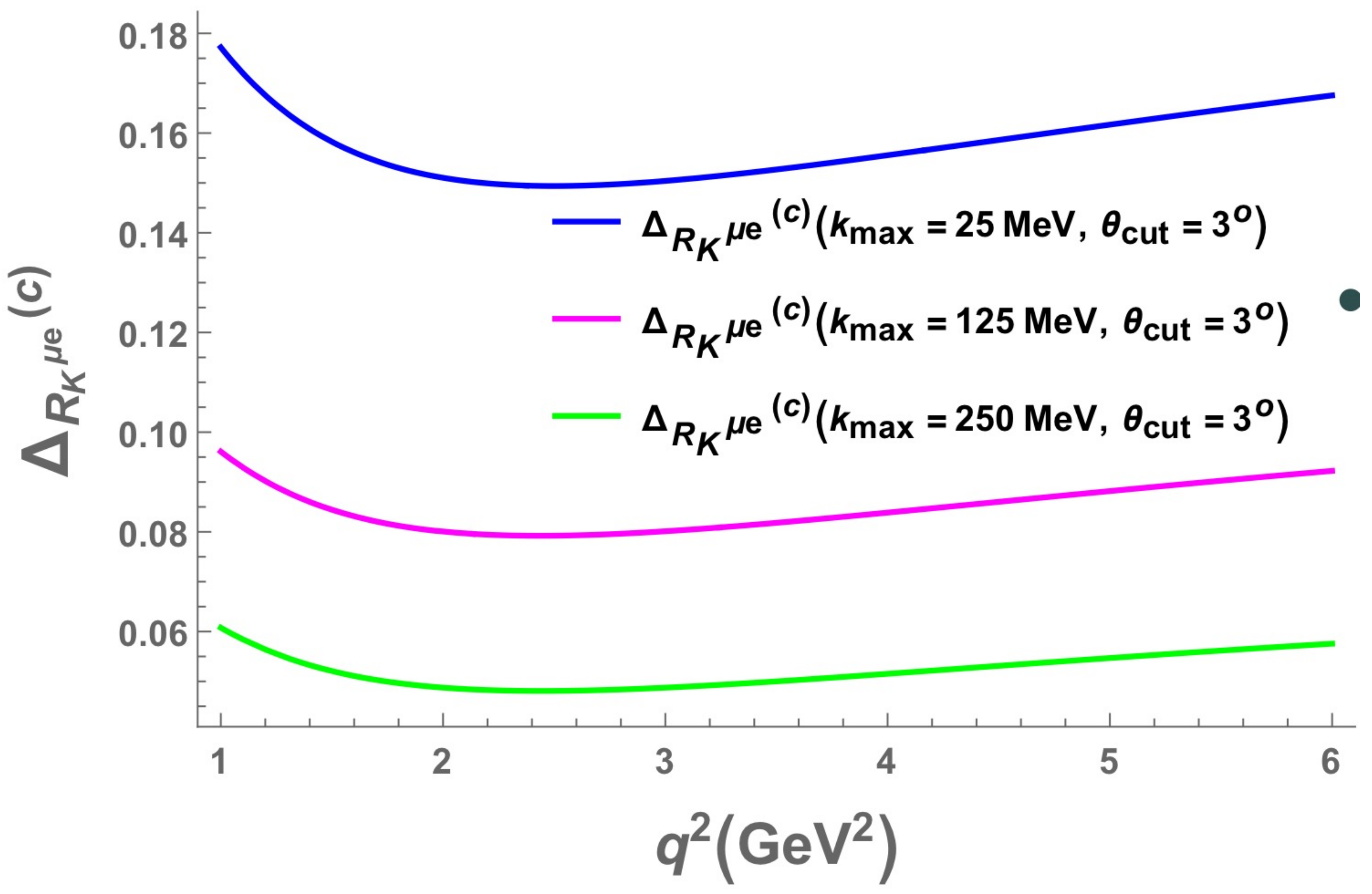
- There is a mild dependence on the photon energy cut k_{max}
- It is sensitive to θ_{cut}, particularly for the case of electrons. Choosing $\theta_{cut} \sim$ few degrees, this sensitivity essentially disappears. For muon, it is not that sensitive.



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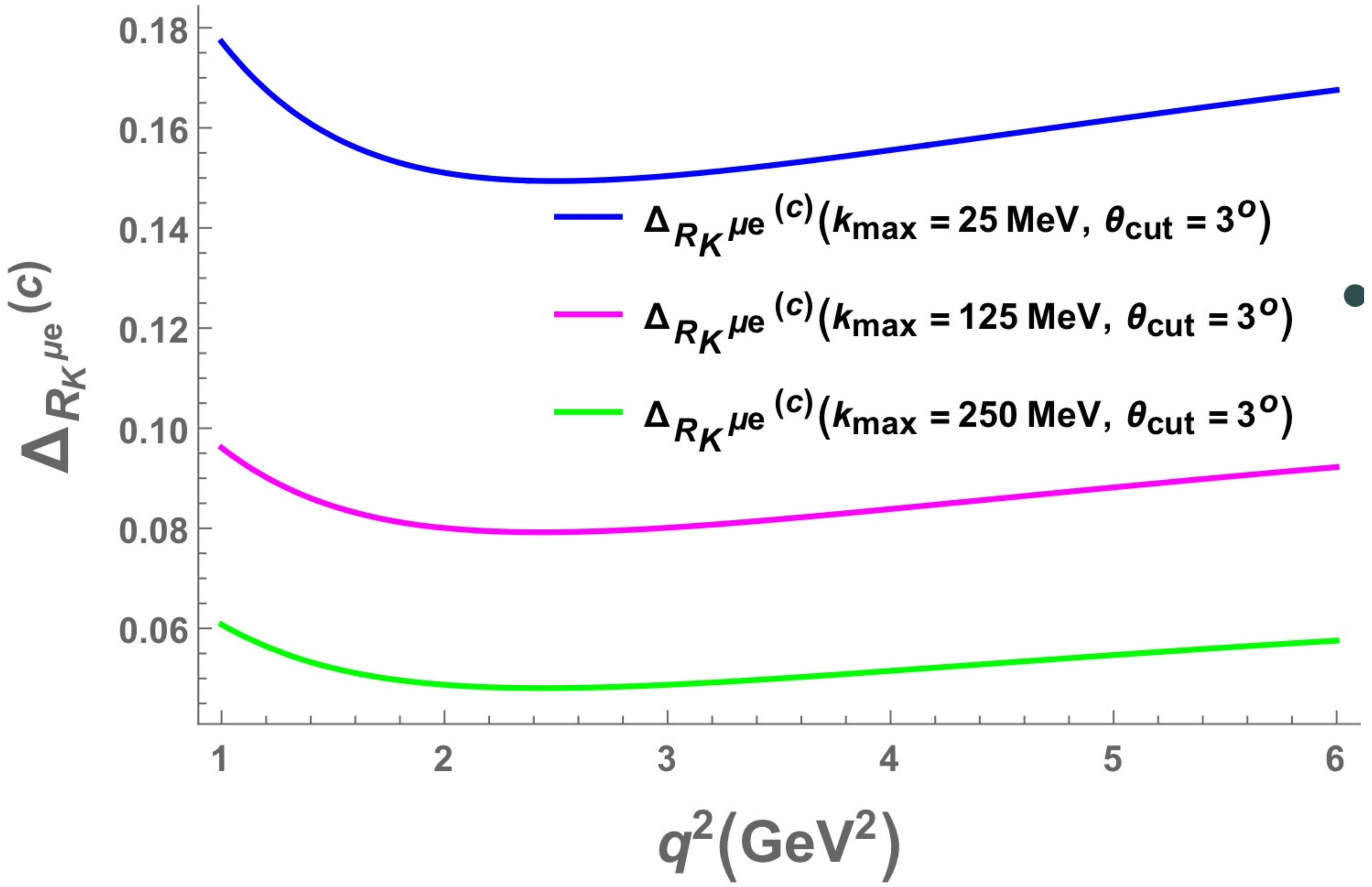

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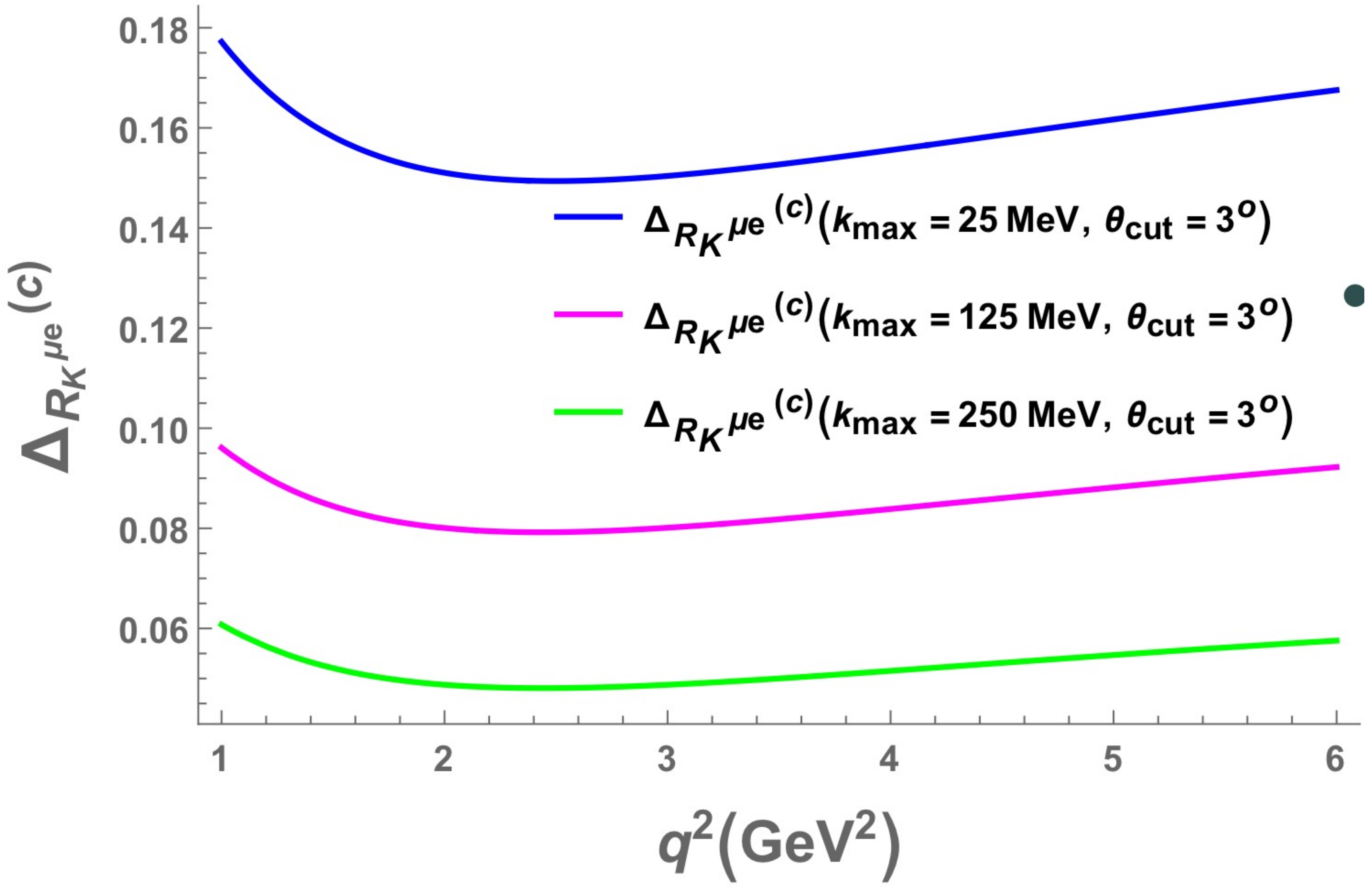

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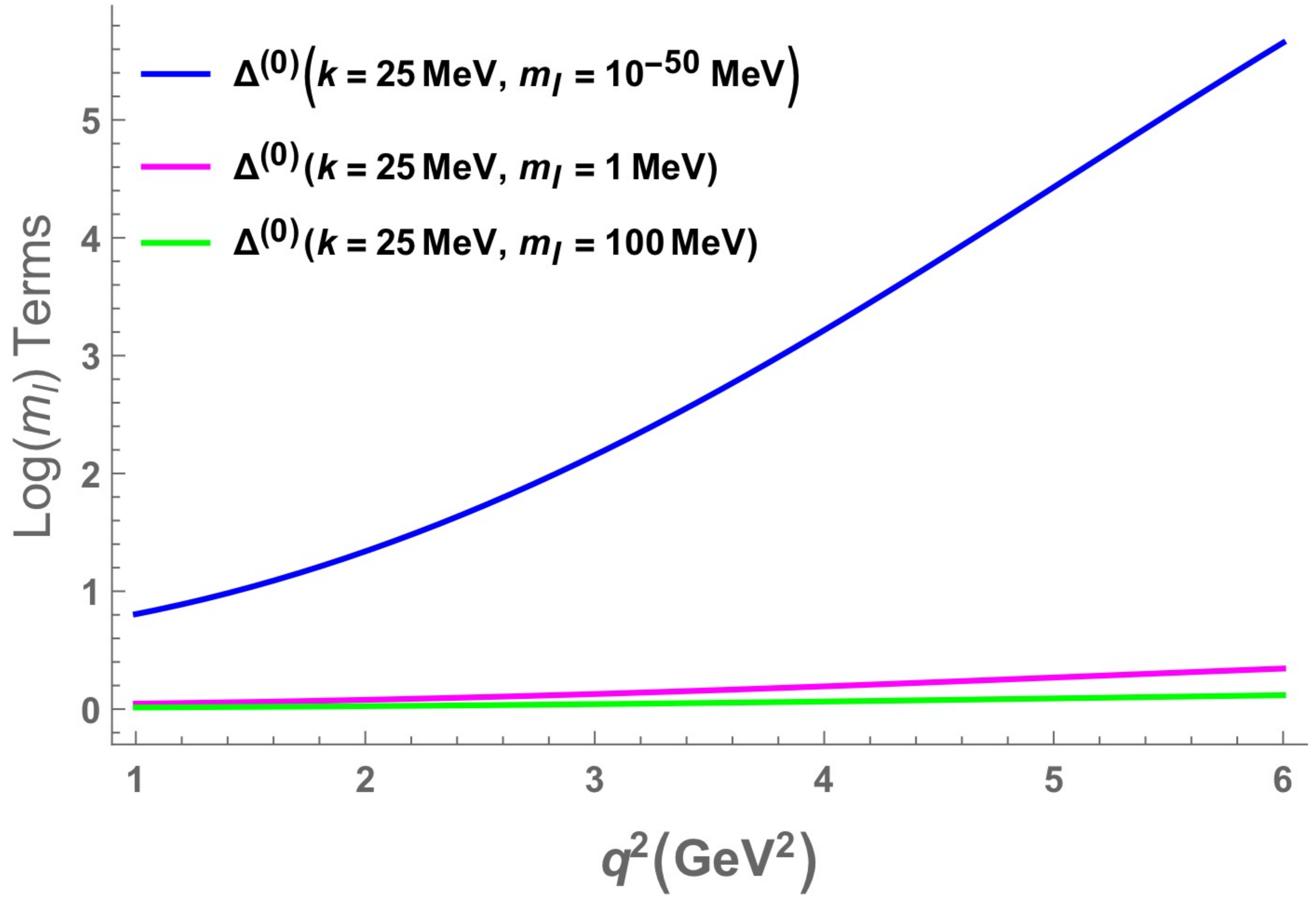

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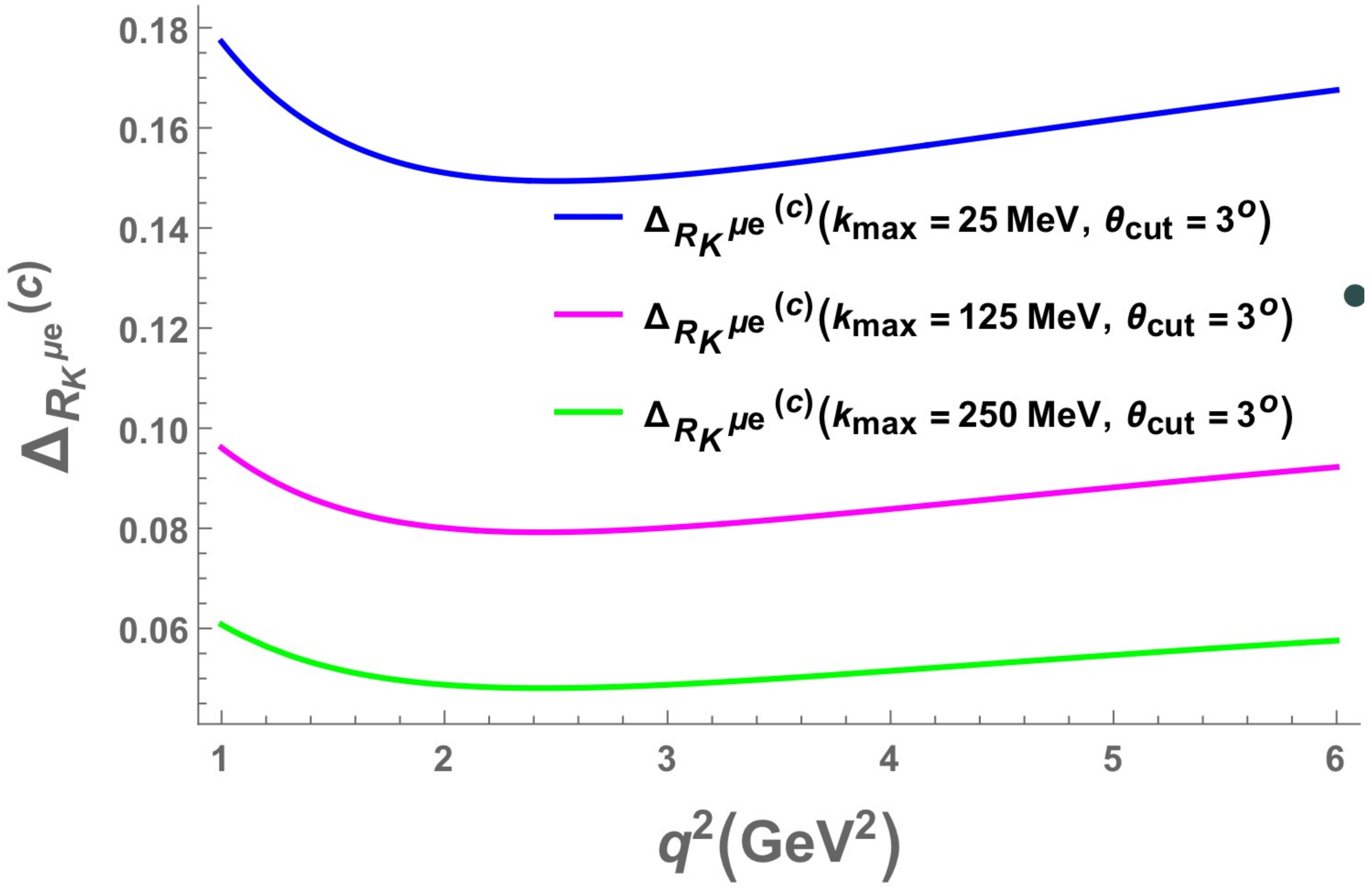

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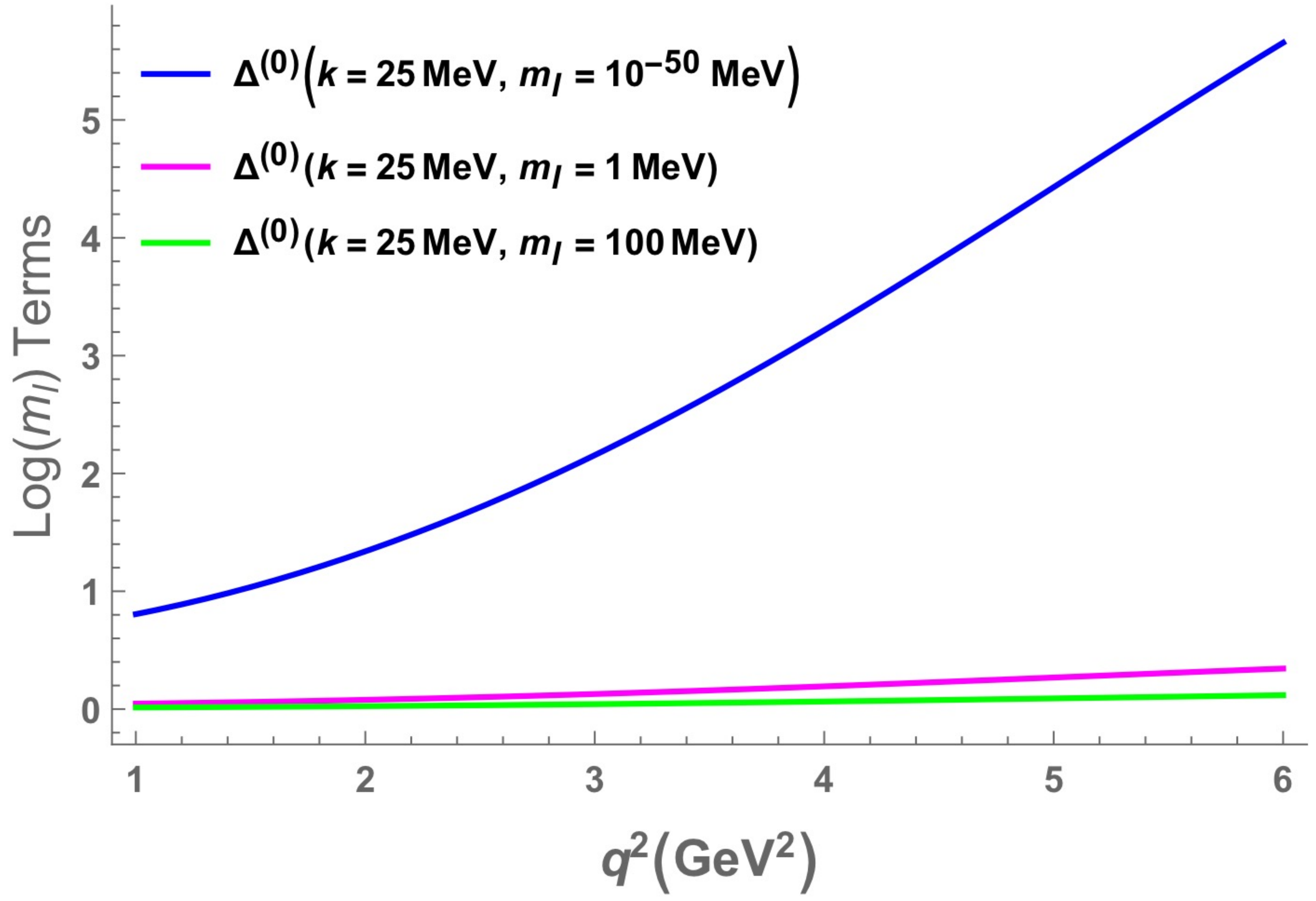

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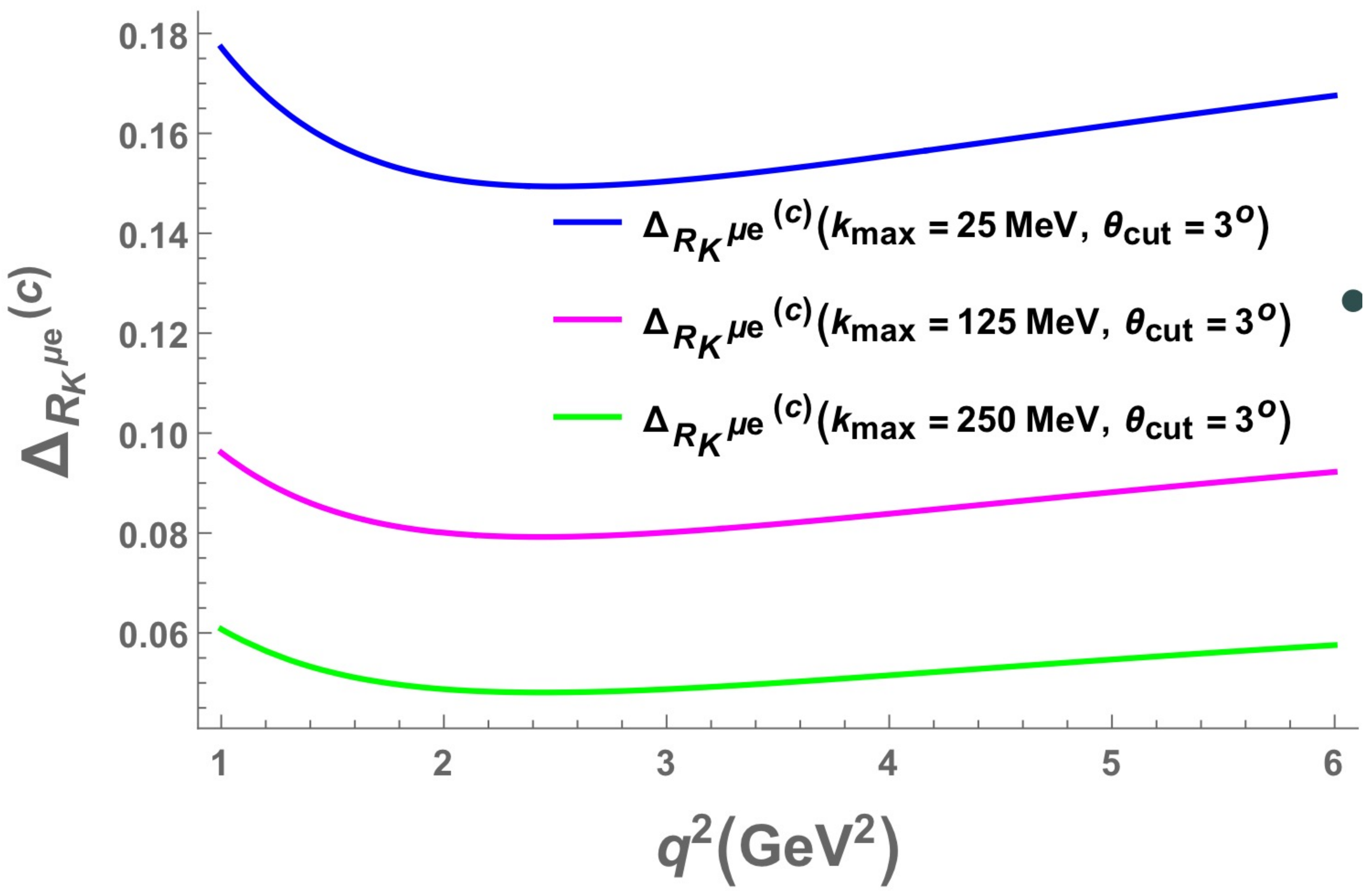
- $\log m_\ell$ terms correspond to hard collinear logs.



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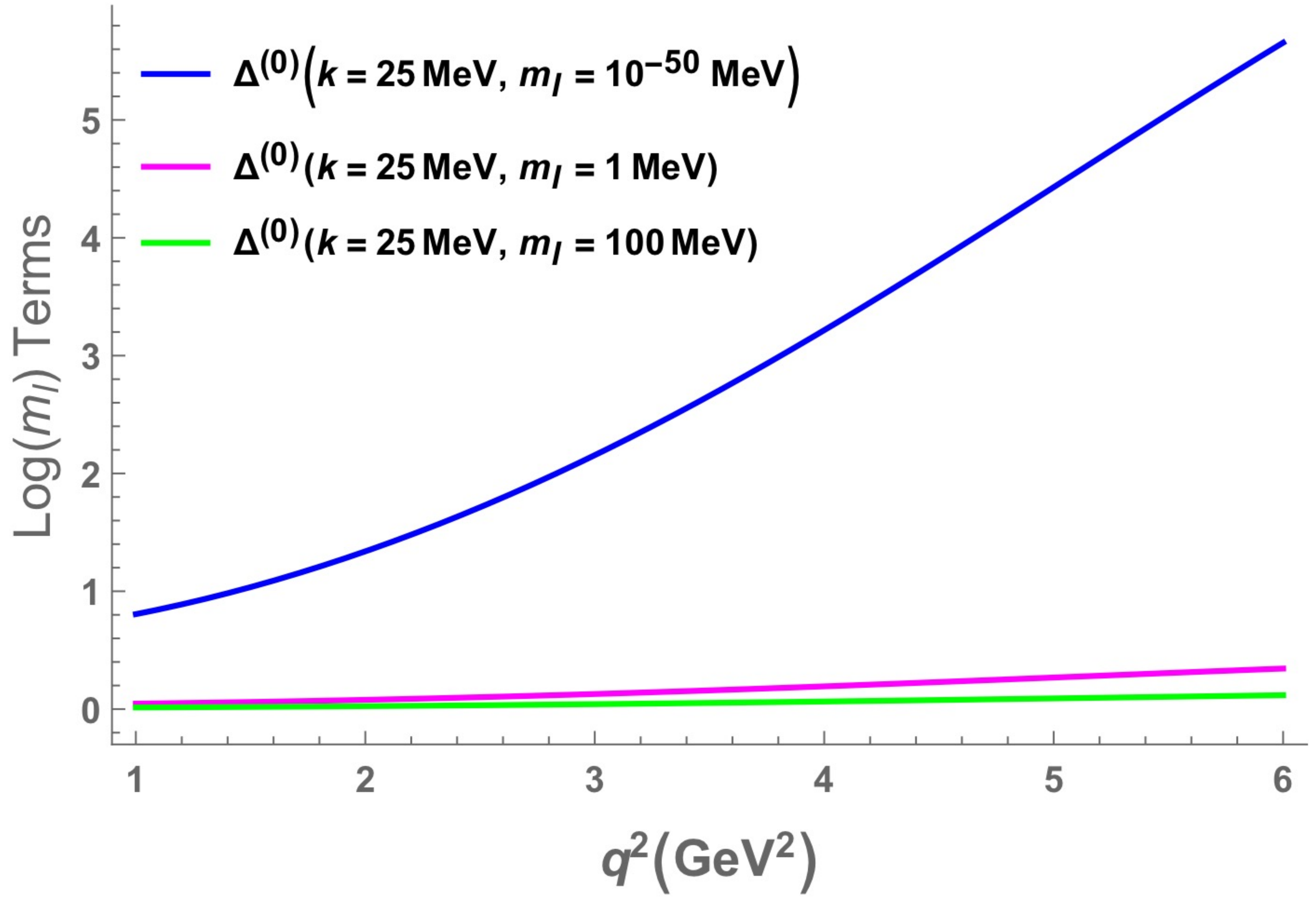

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- We can see the explicit cancellation by choosing a different set of kinematical variables, $t = (p_B - p_k)^2$, $s = (p_k + p_2)^2$, $x = (p_k + k)^2$and $q^2 = (p_2 + p_3)^2$ and E_k in the rest frame of $(q + k)^2$. With a different method results match with G. Isidori et. al.[JHEP 12 (2020) 104].

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Thank you